Universität des Saarlandes
Fachrichtung – Mathematik
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Compact interdisciplinary classes on
"Semigroups generated by integro-differential operators
in Stochastics and Mathematical Physics"

Lecture 01: **Operator semigroups, evolution equations and Markov processes**

The evolution in time of a physical system is in general defined through a system of differential equations and can usually be rewritten as a so called Cauchy problem. So, one is interested in studying well-posedness of the Cauchy problem together with the properties of its solution. This can be done in many important applications via the theory of operator semigroups which yields a solution to the Cauchy problem in terms of a semigroup of linear operators. We outline some connections between operator semigroups and Markov processes. In particular, we discuss Feller processes (including Lévy processes) and corresponding (Feller) semigroups. We establish properties of Feller semigroups and corresponding integro-differential evolution equations (e.g., equations with fractional Laplacians and relativistic Hamiltonians), find connections between convolution semigroups of measures, infinite divisible distributions, Lévy processes and continuous negative definite functions.

Lecture 02: **Construction and approximation of operator semigroups**

Different approaches to construct/approxiamte an operator semigroup will be discussed. We start with some standard procedures, discuss the theorem of Hille and Yosida, then discuss the perturbation techniques, and obtain the variation-of-parameters formula and Dyson-Phillips series representation of a semigroup. Further, we discuss Bernstein functions and subordination of operator semigroups / stochastic processes in the sence of Bochner. We present the Chernoff theorem and its corollaries (Trotter formula and Post–Widder formula). We construct Chernoff approximations of Feller semigroups.

Lecture 03: **Continuous time random walks and anomalous diffusion**

Classical construction of Bernoulli random walks leads (in the scaling limit) to Brownian motion whose probability density function is the fundamental solution of the classical diffusion equation. In the lecture, a more general model of (uncoupled) continuous time random walks (CTRWs) with independent waiting times between independent jumps will be presented; different regimes of CTRWs, leading to different types of diffusion (standard diffusion, subdiffusion, superdiffusion, fractional diffusion) will be discussed. The Montroll–Weiss equation for the probability density of finding the particle (walker) in position x at time t in terms of the joint probability density of jumps and waiting times will be derived. The Montroll–Weiss equation leads to governing equations for probability density functions of the processes obtained as the scaling limits of CTRWs. In the regime of standard diffusion, one obtains the standard diffusion equation. In some particular cases of other regimes of CTRWs, the governing equations are actually time-or/and space-fractional diffusion equations (with Caputo fractional derivative with respect to time or/and fractional Laplacian wih respect to space variables). It is planned to discuss the subordination-type structure of solutions of these equations, to establish relations to operator semigroups and hence to obtain approximations of the solutions.